

Fundamentals of Truth-Functional Logic

PHI 200
(Erion)

Modern truth-functional logic is a very powerful system of deductive logic that utilizes a symbolic notation scheme to represent claims and arguments. At its most fundamental level, truth-functional logic distinguishes *simple claims* from *compound claims*.

simple claim - a claim that does not contain any other claim as a component; in truth-functional logic, simple claims are typically represented with individual letters (P, Q, a, b, etc.)

Ex) P: 'Dr. Erion is silly.'

Ex) Q: 'Dr. Erion is tired.'

Ex) R: 'Dr. Erion is on his way to Canada.'

compound claim - a claim that contains at least one simple claim as a component; in truth-functional logic, compound claims are typically represented by claim variables combined with symbols that signify logical connectives

connective	symbol	read	example	read	approx. meaning	truth table															
negation	~	NOT	~P	'It is not the case that Dr. Erion is silly.'	If P is true, ~P is false; if P is false, ~P is true; switch the truth value	<table style="border-collapse: collapse; margin: auto;"> <tr> <td style="border: none; padding: 0 5px;">P</td> <td style="border: none; padding: 0 5px;">~P</td> </tr> <tr> <td style="border: none; padding: 0 5px;">T</td> <td style="border: none; padding: 0 5px;">F</td> </tr> <tr> <td style="border: none; padding: 0 5px;">F</td> <td style="border: none; padding: 0 5px;">T</td> </tr> </table>	P	~P	T	F	F	T									
P	~P																				
T	F																				
F	T																				
conjunction	&	AND	Q & R	'Dr. Erion is tired and Dr. Erion is on his way to Canada.'	both Q and R are true	<table style="border-collapse: collapse; margin: auto;"> <tr> <td style="border: none; padding: 0 5px;">P</td> <td style="border: none; padding: 0 5px;">Q</td> <td style="border: none; padding: 0 5px;">P & Q</td> </tr> <tr> <td style="border: none; padding: 0 5px;">T</td> <td style="border: none; padding: 0 5px;">T</td> <td style="border: none; padding: 0 5px;">T</td> </tr> <tr> <td style="border: none; padding: 0 5px;">T</td> <td style="border: none; padding: 0 5px;">F</td> <td style="border: none; padding: 0 5px;">F</td> </tr> <tr> <td style="border: none; padding: 0 5px;">F</td> <td style="border: none; padding: 0 5px;">T</td> <td style="border: none; padding: 0 5px;">F</td> </tr> <tr> <td style="border: none; padding: 0 5px;">F</td> <td style="border: none; padding: 0 5px;">F</td> <td style="border: none; padding: 0 5px;">F</td> </tr> </table>	P	Q	P & Q	T	T	T	T	F	F	F	T	F	F	F	F
P	Q	P & Q																			
T	T	T																			
T	F	F																			
F	T	F																			
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disjunction	v	EITHER... OR...	P v Q	'Either Dr. Erion is silly or Dr. Erion is tired.'	either P, or Q, or both, are true	<table style="border-collapse: collapse; margin: auto;"> <tr> <td style="border: none; padding: 0 5px;">P</td> <td style="border: none; padding: 0 5px;">Q</td> <td style="border: none; padding: 0 5px;">P v Q</td> </tr> <tr> <td style="border: none; padding: 0 5px;">T</td> <td style="border: none; padding: 0 5px;">T</td> <td style="border: none; padding: 0 5px;">T</td> </tr> <tr> <td style="border: none; padding: 0 5px;">T</td> <td style="border: none; padding: 0 5px;">F</td> <td style="border: none; padding: 0 5px;">T</td> </tr> <tr> <td style="border: none; padding: 0 5px;">F</td> <td style="border: none; padding: 0 5px;">T</td> <td style="border: none; padding: 0 5px;">T</td> </tr> <tr> <td style="border: none; padding: 0 5px;">F</td> <td style="border: none; padding: 0 5px;">F</td> <td style="border: none; padding: 0 5px;">F</td> </tr> </table>	P	Q	P v Q	T	T	T	T	F	T	F	T	T	F	F	F
P	Q	P v Q																			
T	T	T																			
T	F	T																			
F	T	T																			
F	F	F																			
conditional	→	IF... THEN...	R → P	'If Dr. Erion is on his way to Canada, then Dr. Erion is silly.'	if R is true, then P is also true	<table style="border-collapse: collapse; margin: auto;"> <tr> <td style="border: none; padding: 0 5px;">P</td> <td style="border: none; padding: 0 5px;">Q</td> <td style="border: none; padding: 0 5px;">P → Q</td> </tr> <tr> <td style="border: none; padding: 0 5px;">T</td> <td style="border: none; padding: 0 5px;">T</td> <td style="border: none; padding: 0 5px;">T</td> </tr> <tr> <td style="border: none; padding: 0 5px;">T</td> <td style="border: none; padding: 0 5px;">F</td> <td style="border: none; padding: 0 5px;">F</td> </tr> <tr> <td style="border: none; padding: 0 5px;">F</td> <td style="border: none; padding: 0 5px;">T</td> <td style="border: none; padding: 0 5px;">T</td> </tr> <tr> <td style="border: none; padding: 0 5px;">F</td> <td style="border: none; padding: 0 5px;">F</td> <td style="border: none; padding: 0 5px;">T</td> </tr> </table>	P	Q	P → Q	T	T	T	T	F	F	F	T	T	F	F	T
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